

# The King's Court

Mik Zlatin

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## Intro

In June 2023, I went to the SIAM Conference on Optimization to speak about recent progress towards Woodall's conjecture. I attended many talks at the conference, but by far my favourite was by Professor Lisa Hellerstein on the Stochastic Boolean Function Evaluation problem.

In it, she provided a puzzle with a solution so elegant that I immediately gave the problem as a riddle to all my friends. The following situation is a bit more dramatic than the dry world of boolean functions, but it captures the same mathematical essence.

## Riddle

You are the king of a small province where you preside over a court of  $n$  nobles. Harmony reigns in your land for 10 peaceful years, when one day you hear rumours that the court is plotting a violent coup to overthrow your rule.

You are flabbergasted! Why would the court want to overthrow a benign ruler such as yourself? You decide that you must personally verify whether the rumours are true by interrogating a subset of the court members.

Some court members are more suspicious than others. For each court member  $i \in \{1, \dots, n\}$ , the probability that they are planning to overthrow you is  $p_i$ .

You know that if at least  $k$  court members are plotting to overthrow you, then the coup will happen tomorrow. On the other hand, if fewer than  $k$  court members are plotting to overthrow you, then you are safe.

You scratch your chin and pace about, deep in thought. On the one hand, it is imperative to determine with certainty whether a coup will take place. On the other hand, it would be preferable for the good of the kingdom to keep interrogations to a minimum.

You decide to consult your academic advisor, who is wise in the ways of mathematics:

**Question:** What is the optimal (adaptive) sequence of interrogations so that the king can verify whether a coup will take place using the minimum expected number of interrogations?